"Transform and Variational Methods in Image Processing"

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The series consists of three sections. A preliminary outline of the sections is given below. The references given right after the section titles are intended to provide a somewhat broader introduction to the section topic, or at least –in the case of the papers [6, 7]– to be representative for the central topic of the section.
The references given after the title of a talk are directly relevant to the subject of the talk. I refer to my papers and scripts chiefly because they are freely available.

I. *Wavelet orthonormal bases: Construction, properties, algorithms [17, Chapter VII], [12, Chapters 3 and 4]*

   I.1 Multiresolution analysis and wavelets [5, 12, 17, 20]

   I.2 Compactly supported wavelets [5, 12, 17, 20]

   I.3 The fast wavelet transform and its filterbank interpretation [5, 12, 17, 20]

II. *Wavelet algorithms as a special case of variational image processing [6, 7, 3]*

   II.1 Variational image models and function spaces [3, 19]

   II.2 Wavelet characterization of function spaces: Besov spaces [20, 18], BV [4]

   II.3 Wavelet shrinkage denoising and its variational interpretation [11, 6]

   II.4 Variations on wavelet shrinkage, and relations to the Osher-Rudin-Fatemi model [16]

III. *Multiscale image approximation techniques beyond wavelets [13]*

   III.1 Curvelets: A filterbank approach with improved directional resolution [1, 2, 8, 9]

   III.2 Wedgelets and related approximation schemes [10, 14, 15]

A brief description of the three sections and their aims is as follows:

I. This section deals with the construction of wavelet orthonormal bases in one and two dimensions. The central notion is that of *multiresolution analysis*, which plays a vital role both in the construction of wavelet orthonormal bases and in providing fast decomposition algorithms. For the purposes of section II, the most important issues are additional properties of wavelets, such as compact supports, smoothness and vanishing moments. Moreover, we present the filterbank interpretation of the fast wavelet transform that provides a natural approach to generalizations presented in Section III.

II. In this section we consider variational aspects of wavelet algorithms. We start out by giving a review of regularization approaches in image processing, which typically rely on the choice of a suitable Banach function space as model space. We then proceed to present wavelet characterizations of some of these spaces, and show how certain regularization problems relate to algorithms such as wavelet shrinkage. We then sketch more recent (and sophisticated) algorithms for denoising.

III. Based on the observation that wavelet orthonormal bases are not ideally suited for the approximation of piecewise smooth images with smooth boundaries, several alternative constructions have been proposed recently. We present two approaches in more details, curvelets and wedgelets.

Pdf-files of [6, 7, 12, 13] are provided. For all other references, with the exception of the books [3, 5, 17, 18, 19] and the paper [4], internet locations for preprint versions are indicated.
There are also two presentations in .pdf-format,

- *Wavelet analysis of discrete-time and continuous-time signals*, related to I and II.2
- *Wedgelets and related schemes: Multiscale modelling of geometric image structures*, related to III.2

which can be found under http://ibb.gsf.de/homepage/hartmut.fuehr/talks.html

Literatur

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